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Minus Dominating and Total Minus Dominating Functions of Corona Product Graph of a Path with a Star

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Abstract: Graph theory is one of the most flourishing branches of modern mathematics and computer applications. Domination in graphs has been studied extensively in recent years and it is an important branch of graph theory. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [6,7]. Recently dominating functions in domination theory have received much attention. In this paper we present some results on minimal minus dominating and total minus dominating functions of corona product graph of path with a star.

Index Terms: Corona Product, Path, Star, minus dominating function, Total minus dominating function.

1. INTRODUCTION

Domination Theory has a wide range of applications to many fields like Engineering, Communication Networks, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[1], Cockayne, E.J. and Hedetniemi, S.T. [2] have studied various domination parameters of graphs.

The minus dominating function was defined by Dunbar et al. [3]. Zelinka [12] gave a lower bound of a minus domination number for a cubic graph and Dunbar et al.[3] did the same work for regular graphs. The efficient minus domination problem has applications in sociology, electronics and facility location of operation research. The total minus dominating function of a graph has been defined by Harris, L. and Hattingh, J.H. [5].

Frucht and Harary [4] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2 .

The authors have studied some dominating functions of corona product graph of a cycle with a complete graph [8] and published papers on minimal dominating functions, some variations of Y – dominating functions and Y – total dominating functions [9,10,11]. In this paper we discuss some results on minus dominating functions and total minus dominating functions of the graph $G = P_n \odot K_{1,m}$.

2. CORONA PRODUCT OF P_n AND $K_{1,m}$

The **corona product** of a path P_n with star $K_{1,m}$ is a graph obtained by taking one copy of a n – vertex path P_n and n copies of $K_{1,m}$ and then joining the i^{th} vertex of P_n to every vertex of i^{th} copy of $K_{1,m}$ and it is denoted by $P_n \odot K_{1,m}$.

We require the following theorem whose proof can be found in Siva Parvathi, M. [8].

Theorem 2.1: The degree of a vertex v_i in $G = P_n \odot K_{1,m}$ is given by

$$d(v_i) = \begin{cases} m+3, & if \quad v_i \in P_n \text{ and } 2 \le i \le (n-1), \\ m+2, & if \quad v_i \in P_n \text{ and } \quad i = 1 \text{ or } n, \\ m+1, & if \quad v_i \in K_{1,m} \text{ and } v_i \text{ is in first partition}, \\ 2, & if \quad v_i \in K_{1,m} \text{ and } v_i \text{ is in second partition}. \end{cases}$$

3. Minus Dominating Functions

In this section we discuss the concepts of minus dominating functions and efficient minus domination functions of the corona product graph $P_n \odot K_{1,m}$ and some results on these functions are obtained. Let us recall the definitions of these functions.

Definition: Let $G(V, E)$ be a graph. A function $f : V \to \{-1, 0, 1\}$ is called a **minus dominating function** (Minus DF) of G if

$$f(N[v]) = \sum_{u \in N[v]} f(u) \ge 1, \; for\ each \quad v \in V.$$

A minus dominating function f of G is called a **minimal minus dominating function** if for all $g < f$, g is not a minus dominating function.

Definition: Let $G(V, E)$ be a graph. A function $f : V \to \{-1, 0, 1\}$ is called an **efficient minus dominating**

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function (E Minus DF) of G if

$$f(N[v]) = \sum_{u\in N[v]} f(u) = 1, \ for\ each\ \ v \in V.$$

Theorem 3.1: A function $f : V \rightarrow \{-1, 0, 1\}$ defined by

$$f(v) = \begin{cases} -1, & \text{for one vertex v in each copy of } K_{1,m} \text{ whose degree is 2 in G,} \\ 0, & \text{for other vertices in each copy of } K_{1,m} \text{ whose degree is 2 in G,} \\ 1, & \text{otherwise.} \end{cases}$$

is a minimal minus dominating function of $G = P_n \odot K_{1,m}$.

Proof: Let f be a function defined as in the hypothesis.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Then $N[v]$ contains $m + 1$ vertices of $K_{1,m}$ and three vertices of P_n in G.

So $$\sum_{u\in N[v]} f(u) = 1+1+1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 3.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Then $N[v]$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of P_n in G.

So $$\sum_{u\in N[v]} f(u) = 1+1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 2.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N[v]$ contains $m + 1$ vertices of $K_{1,m}$ and one vertex of P_n in G.

So $$\sum_{u\in N[v]} f(u) = 1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 1.$$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $f(v) = -1$ or $f(v) = 0$.

Now $N[v]$ contains two vertices of $K_{1,m}$ and one vertex of P_n in G.

If $f(v) = -1$, then $\sum_{u\in N[v]} f(u) = 1+1+(-1) = 1.$

If $f(v) = 0$, then $\sum_{u\in N[v]} f(u) = 1+1+0 = 2.$

Therefore for all possibilities, we get

$$\sum_{u\in N[v]} f(u) \geq 1, \ \forall \ \ v \in V.$$

This implies that f is a Minus DF.

Now we check for the minimality of f.

Define $g : V \rightarrow \{ -1, 0, 1 \}$ by

$$g(v) = \begin{cases} -1, & \text{for one vertex v in each copy of } K_{1,m} \text{ whose degree is 2 in G} \\ & \text{and for any one vertex } v_k \in P_n \text{ in G,} \\ 0, & \text{for other vertices in each copy of } K_{1,m} \text{ whose degree is 2 in G,} \\ 1, & \text{otherwise.} \end{cases}$$

Since strict inequality holds at the vertex $v_k \in V$, it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then

$$\sum_{u\in N[v]} g(u) = (-1)+1+1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 1.$$

Sub case 2: Let $v_k \notin N[v]$.

Then

$$\sum_{u\in N[v]} g(u) = 1+1+1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 3.$$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then

$$\sum_{u\in N[v]} g(u) = (-1)+1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 0.$$

Sub case 2: Let $v_k \notin N[v]$.

Then $$\sum_{u\in N[v]} g(u) = 1+1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 2.$$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u\in N[v]} g(u) = (-1)+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = -1.$$

Sub case 2: Let $v_k \notin N[v]$.

Then $$\sum_{u\in N[v]} g(u) = 1+\left[1+(-1)+\underbrace{0+.......+0}_{(m-1)-times}\right] = 1.$$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $g(v) = -1$ or $g(v) = 0$.

Sub case 1: Let $v_k \in N[v]$.

If $g(v) = -1$, then $\sum_{u\in N[v]} g(u) = (-1)+1+(-1) = -1.$

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If $g(v) = 0$, then $\sum_{u \in N[v]} g(u) = (-1) + 1 + 0 = 0.$

Sub case 2: Let $v_k \notin N[v]$.

If $g(v) = -1$, then $\sum_{u \in N[v]} g(u) = 1 + 1 + (-1) = 1.$

If $g(v) = 0$, then $\sum_{u \in N[v]} g(u) = 1 + 1 + 0 = 2.$

This implies that $\sum_{u \in N[v]} g(u) < 1$, for some $v \in V$.

So g is not a Minus DF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a Minus DF.

Thus f is a minimal Minus DF.

Theorem 3.2: A function $f : V \to \{-1, 0, 1\}$ defined by

$$f(v) = \begin{cases} 1, & \text{for } v \text{ whose degree is } m+1 \text{ in each copy of } K_{1,m} \text{ in } G, \\ 0, & \text{otherwise.} \end{cases}$$

is an efficient minus dominating function of $G = P_n \odot K_{1,m}$.

Proof: Consider the graph $G = P_n \odot K_{1,m}$ with vertex set V.

Let f be a function defined as in the hypothesis.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

$$\text{So } \sum_{u \in N[v]} f(u) = 0 + 0 + 0 + \left[1 + \underbrace{0 + \dots\dots + 0}_{m-times} \right] = 1.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

$$\text{So } \sum_{u \in N[v]} f(u) = 0 + 0 + \left[1 + \underbrace{0 + \dots\dots + 0}_{m-times} \right] = 1.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

$$\text{So } \sum_{u \in N[v]} f(u) = 0 + \left[1 + \underbrace{0 + \dots\dots + 0}_{m-times} \right] = 1.$$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

$$\text{So } \sum_{u \in N[v]} f(u) = 0 + 1 + 0 = 1.$$

Therefore for all possibilities,

we get $\sum_{u \in N[v]} f(u) = 1, \quad \forall \quad v \in V.$

This implies that f is an efficient minus dominating function.

4. Total Minus Dominating Functions

In this section we discuss about total minus dominating functions of corona product graph $G = P_n \odot K_{1,m}$ and results on total minus dominating functions of this graph are presented. First let us recall some definitions.

Definition: Let $G(V, E)$ be a graph. A function $f : V \to \{-1, 0, 1\}$ is called a **total minus dominating function** (T Minus DF) of G if

$$f(N(v)) = \sum_{u \in N(v)} f(u) \geq 1, \ \text{for each } v \in V.$$

A total minus dominating function f of G is called a **minimal total minus dominating function** if for all $g < f$, g is not a total minus dominating function.

Theorem 4.1: A function $f : V \to \{-1, 0, 1\}$ defined by

$$f(v) = \begin{cases} 1, & \text{for the vertices of } P_n \text{ in } G, \\ 0, & \text{otherwise.} \end{cases}$$

is a Minimal Total Minus Dominating Function of $G = P_n \odot K_{1,m}$.

Proof: Consider the graph $G = P_n \odot K_{1,m}$ with vertex set V.

Let f be a function defined as in the hypothesis.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of P_n in G.

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{(m+1)-times} \right] = 2.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and one vertex of P_n in G.

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{(m+1)-times} \right] = 1.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N(v)$ contains m vertices of $K_{1,m}$ whose degree is 2 and one vertex of P_n in G.

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{m-times} \right] = 1.$$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $N(v)$ contains one vertex of $K_{1,m}$ whose degree is $m + 1$ and one vertex of P_n in G.

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So $\sum_{u \in N(v)} f(u) = 1 + 0 = 1.$

Therefore for all possibilities,

we get $\sum_{u \in N(v)} f(u) \geq 1, \quad \forall \quad v \in V.$

This implies that f is a Total Minus DF.

Now we check for the minimality of f.

Define g : V → { -1, 0, 1} by

$$g(v) = \begin{cases} -1, \text{ if } v = v_k \in P_n \text{ in } G, \\ 1, \text{ if } v \in P_n - \{v_k\} \text{ in } G, \\ 0, \text{ otherwise.} \end{cases}$$

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = (-1) + 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{(m+1)-times} \right] = 0.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{(m+1)-times} \right] = 2.$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = (-1) + \left[\underbrace{0 + 0 + \dots\dots + 0}_{(m+1)-times} \right] = -1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{(m+1)-times} \right] = 1.$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = (-1) + \left[\underbrace{0 + 0 + \dots\dots + 0}_{m-times} \right] = -1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + \left[\underbrace{0 + 0 + \dots\dots + 0}_{m-times} \right] = 1.$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = (-1) + 0 = -1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + 0 = 1.$

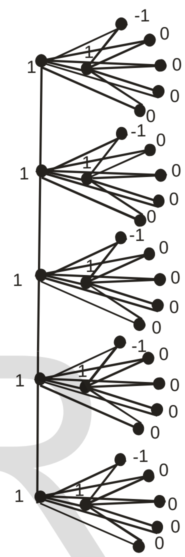
This implies that $\sum_{u \in N(v)} g(u) < 1$, for some v ∈ V.

So g is not a Total Minus DF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a T Minus DF.

Thus f is a minimal T Minus DF.

5. ILLUSTRATION



$G = P_5 \odot K_{1,5}$

The function 'f' takes the value -1 for one vertex in each copy of $K_{1,5}$ whose degree is 2, the value 0 for other vertices in each copy of $K_{1,5}$ whose degree is 2 and the value 1 for the vertices of P_5 and the remaining vertex in $K_{1,5}$.

6. CONCLUSION

It is interesting to study the convexity of minimal dominating and total dominating functions of corona product graph of a cycle with a complete graph. This work gives the scope for an extensive study of dominating functions in general of these graphs.

6. REFERENCES

[1] Haynes, T.W., Hedetniemi, S.T. and Slater, P.J. - Domination in Graphs: Advanced Topics, Marcel Dekker, Inc., New York, (1998).

[2] Haynes, T.W., Hedetniemi, S.T. and Slater, P.J. Fundamentals of domination in graphs, Marcel Dekker, Inc., New York , (1998).

[3] Allan, R.B. and Laskar, R.C. – On domination, independent domination numbers of a graph. Discrete Math., 23 (1978), 73 – 76.

[4] Cockayne, E.J. and Hedetniemi, S.T. - Towards a theory of domination in graphs. Networks, 7 (1977), 247 – 261.

[5] Cockayne, C.J., Dawes, R.M. and Hedetniemi, S.T- Total domination in graphs, Networks, 10 (1980), 211 – 219.

ISSN 2229-5518
[6] Jeelani Begum, S. - Some studies on dominating functions of Quadratic Residue Cayley Graphs, Ph. D. thesis, Sri Padmavathi Mahila Visvavidyalayam, Tirupati, Andhra Pradesh, India, (2011).
[7] Frucht, R. and Harary, F. - On the corona of Two Graphs, Aequationes Mathematicae, Volume 4, Issue 3 (1970), 322 – 325.
[8] Siva Parvathi, M – Some studies on dominating functions of corona product graphs, Ph.D thesis, Sri Padmavati Mahila Visvavidyalayam, Tirupati, Andhra Pradesh, India, (2013).
[9] Cockayne, E.J., Mynhardt, C.M. and Yu, B. Total dominating functions in trees: Minimality and Convexity, Journal of Graph Theory, 19(1995), 83 – 92.
[10] Yu, B., Convexity of minimal total dominating functions in graphs, J. Graph Theory, 24 (4) (1997), 313 – 321.
[11] Reji Kumar, K., Studies in Graph Theory – Dominating functions, Ph.D. thesis, Manonmaniam Sundaranar University, Tirunelveli, India, (2004).
[12] Cockayne, E.J. and Mynhardt, C.M. Convexity of extremal domination – related functions of graphs. In: Domination in Graphs – Advaned Topics, (Ed.T.W.Haynes, S.T. Hedetniemi, P.J. Slater), Marcel Dekker, New York, (1998), 109 – 131.